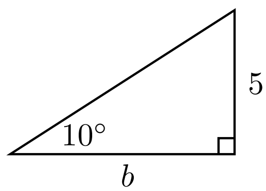
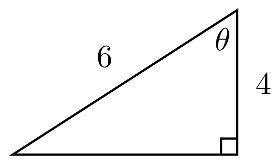
No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

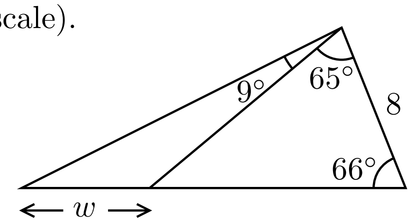
Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(6 pts). Solve for b and θ in the figures (not to scale).





2(14 pts). Find the distance w in the figure (not to scale).



3(7 pts). An airplane flying horizontally at an altitude of $20,000$ ft approaches an observer atop a 1,000-ft-high hill. At one moment, the line of sight from the observer to the plane makes a $41°$ angle with the horizontal. At that moment, what is the distance (in feet) between the plane and the observer?

4(7 pts). Points A and B are on opposite sides of Lake Jake. From a third point C, the angle between the lines of sight to A and to B is $70°$. If AC is 3 km long and BC is 2 km long, find AB (in km).

5(19 pts). Evaluate the expression.

a. $\log_4(16)$ b. $\log_{81}(\frac{1}{3})$ c. $\ln\left(\frac{e^3}{e^4e^6}\right)$ d. $10^{\log 13}$ e. $25^{\log_5 3}$

6(15 pts). Find the domain, all intercepts, and (the equations of) all asymptotes for the given function. Sketch the function's graph.

a. $g(x) = \log_4(x + 16)$ b. $f(x) = -9 + 3^{x-2}$

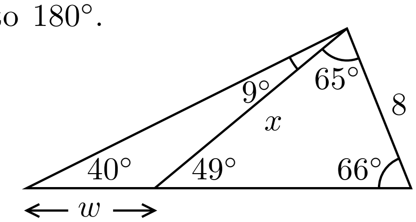
7(9 pts). Carbon 14 (C-14) is a radioactive substance found in all living things on Earth. Analysis of a bone fragment shows that it contains 20% of the C-14 that it contained when it was living tissue. If the half-life of C-14 is 5730 years, how many years old is the bone fragment?

8(23 pts). Solve the following equations.

a. $3^{-2x} = 5^{1+x}$ b. $\log_5|1 - v| = 2$ c. $2^{2p} - 12 \cdot 2^p + 32 = 0$

1.(Source: 4.10,11,15) $\frac{5}{b} = \tan 10°$, so $b = \frac{5}{\tan 10°}$. (This also equals $5 \cot 10° = 5 \tan 80°$.) $\cos\theta = \frac{4}{6} = \frac{2}{3}$. As an acute angle, θ is in $[0, 180°]$, and therefore $\theta = \cos^{-1}(\frac{2}{3})$.

2.(Source: 4.12.21) First find the other angles shown in the figure below from the fact that angles in a triangle must sum to 180°. Then find the side x by the Law of Sines:

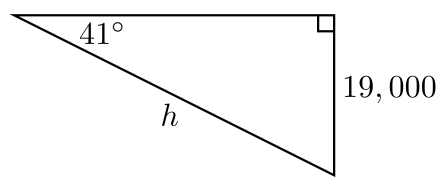


$$\frac{x}{\sin 66°} = \frac{8}{\sin 49°} \quad \Longrightarrow \quad x = \frac{8 \sin 66°}{\sin 49°}$$

Then Law of Sines again:

$$\frac{w}{\sin 9°} = \frac{x}{\sin 40°} \quad \Longrightarrow \quad w = \frac{x \sin 9°}{\sin 40°} = \frac{8 \sin 66° \sin 9°}{\sin 49° \sin 40°}$$

3.(Source: 4.11.8) The desired distance is the length of the hypotenuse of this triangle, which we find with right-triangle trigonometry:



$$\frac{19000}{h} = \sin 41° \quad \Longrightarrow \quad h = \frac{19000}{\sin 41°} = 19000 \csc 41°.$$

4.(Source: 4.13.more.3) From the Law of Cosines,

$$AB = \sqrt{AC^2 + BC^2 - 2\, AC\, BC \cos 70°} = \sqrt{13 - 12 \cos 70°}.$$

5. It is essential to remember the properties of logs, which you'll find at
`http://kunklet.people.cofc.edu/MATH111/zill0502prob.pdf`

5a.(Source: 5.2.14) $\log_4(16) = 2$ because $4^2 = 16$.

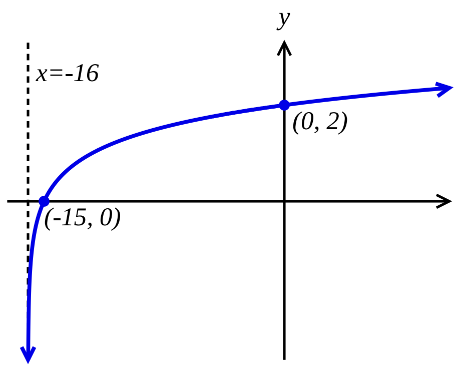
5b.(Source: 5.2.16) It might help to rewrite $\log_{81}(\frac{1}{3}) = x$ in exponential form: $81^x = \frac{1}{3}$, which is, $(3^4)^x = 3^{4x} = 3^{-1}$, so $4x = -1$, and therefore $\log_{81}(\frac{1}{3}) = -\frac{1}{4}$.

5c.(Source: 5.2.18) $\ln\left(\frac{e^3}{e^4 e^6}\right) = \ln(e^{3-4-6}) = \ln(e^{-7}) = -7$.

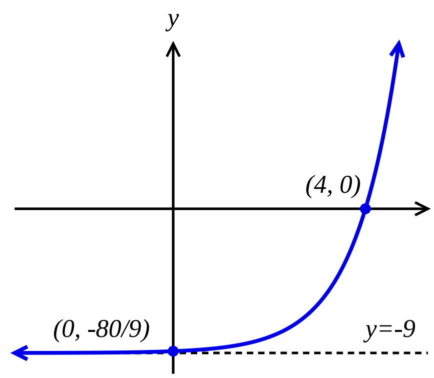
5d.(Source: 5.2.21) "log" with no base means "$\log_{10}$," and so $10^{\log 13} = 13$.

5e.(Source: 5.2.20) $25^{\log_5 3} = (5^2)^{\log_5 3} = 5^{2 \log_5 3} = 5^{\log_5 3^2} = 3^2 = 9$.

6a.(Source: 5.2.28,40) The graph of $g(x) = \log_4(x + 16)$ is obtained by shifting the usual logarithm curve 16 units to the left. That moves the vertical asymptote to $x = -16$. Since the logarithm requires that $x + 16 > 0$, the domain of $g(x)$ is $(-16, \infty)$. At $x = 0$, calculate $y = 2$ (as in Problem 5a). Set $y = 0$ and solve to find the x-intercept: $\log_4(x + 16) = 0 \Rightarrow x + 16 = 4^0 = 1 \Rightarrow x = -15$.



a. $y = \log_4(x + 16)$



b. $y = -9 + 3^{x-2}$

6b.(Source: 5.1.11) The graph of $f(x) = -9 + 3^{x-2}$ is obtained by shifting the graph of the increasing exponential function 3^x 2 units right and 9 units down. (The shift to the right is hard to represent in a drawing.) That moves the horizontal asymptote to $y = -9$. The domain is all real numbers, $(-\infty, \infty)$. At $x = 0$, calculate $y = -9 + 3^{-2} = -9 + \frac{1}{9} = -\frac{80}{9}$. When $y = 0$, solve to find the x-intercept: $3^{x-2} = 9 = 3^2 \Rightarrow x - 2 = 2 \Rightarrow x = 4$.

7.(Source: 5.4.18) Let $y = y_0 e^{kt}$ denote the amount of C-14 in the bone t years after its death. Although we don't know the initial amount y_0, we can solve for k using the given half-life:

$$\tfrac{1}{2} y_0 = y_0 e^{5730k} \Rightarrow \tfrac{1}{2} = e^{5730k} \Rightarrow \ln(\tfrac{1}{2}) = 5730k \Rightarrow \tfrac{1}{5730} \ln(\tfrac{1}{2}) = k$$

To find the current age of the bone, set $y = 0.20 y_0$ and solve for t:

$$\tfrac{1}{5} y_0 = y_0 e^{tk} \Rightarrow \tfrac{1}{5} = e^{tk} \Rightarrow tk = \ln(\tfrac{1}{5}) \Rightarrow t = \tfrac{1}{k} \ln(\tfrac{1}{5}) = \frac{5730 \ln(\frac{1}{5})}{\ln(\frac{1}{2})} \left(= \frac{5730 \ln 5}{\ln 2} \right)$$

8a.(Source: 5.3.19) To get the variable out of the exponents, take a logarithm of both sides:

$$\ln(3^{-2x}) = \ln(5^{1+x}) \Rightarrow -2x \ln 3 = (1 + x) \ln 5.$$

Now distribute the $\ln 5$ and collect like terms:

$$\begin{aligned} -2x \ln 3 &= \ln 5 + x \ln 5 \\ -\ln 5 &= 2x \ln 3 + x \ln 5 = x(2 \ln 3 + \ln 5) \end{aligned} \quad \Longrightarrow \quad x = \frac{-\ln 5}{2 \ln 3 + \ln 5}$$

8b.(Source: 5.3.15,17,28) Rewrite in exponential form: $|1 - v| = 5^2 = 25$. Now rewrite the equation in equivalent form without the absolute value symbols, and solve: $1 - v = \pm 25 \Rightarrow v = 1 \mp 25 = 26$ or -24.

8c.(Source: 5.3.41) To solve, it might help to create a new variable $s = 2^p$ and to note that $2^{2p} = s^2$. This results in a quadratic equation, which we solve by factoring:

$$0 = s^2 - 12s + 32 = (s - 4)(s - 8) \Rightarrow s = 4 \text{ or } 8.$$

Now revert to the original variable p:

$$2^p = 4 = 2^2 \text{ or } 8 = 2^3 \Rightarrow p = 2 \text{ or } 3.$$