No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(20 pts). Find the values of the given trig functions at the given values of t. Write "DNE" when appropriate. Supporting work not required on this problem.

t	0	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$-\frac{17\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{17\pi}{2}$
$\sin t$						
$\cos t$						
$\tan t$						
$\cot t$						
$\sec t$						
$\csc t$						

2(3 pts). Convert $-64°$ to radians.

3(3 pts). Find the arclength subtended by a central angle of 2 radians in a circle of radius 3 cm.

4(26 pts). Find the following.

a. $\cos\left(\frac{3\pi}{8}\right)$ b. $\sin\left(-\frac{5\pi}{12}\right)$ c. $\tan^{-1}(\sqrt{3})$

d. $\sin^{-1}\left(-\frac{1}{2}\right)$ e. $\sin\left(\tan^{-1}(2)\right)$ f. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$

5(13 pts). Sketch one cycle of the graph of $y = 2 - \cos\left(3x - \frac{\pi}{4}\right)$. Draw the axes where you wish and label hashmarks so as to clearly indicate every point in your cycle where the cosine equals 0, 1, or -1. What is the amplitude of this function?

6(9 pts). Sketch one cycle of the graph of $y = 2\sec\left(3x - \frac{\pi}{4}\right)$. Draw the axes where you wish and label hashmarks so as to clearly indicate every point in your cycle where the secant equals 1 or -1. Give the equation(s) of any asymptotes that occur in your cycle. What is the period of this function?

7(8 pts). Suppose that $\sin x = -\frac{1}{3}$ and that $\frac{\pi}{2} < x < \frac{3\pi}{2}$. Find the values of the other five trig functions at x.

8(5 pts). Use the substitution $x = b\tan\theta$, $-\pi/2 < \theta < \pi/2$ to rewrite the algebraic expression $\sqrt{b^2 + x^2}$ as a trigonometric expression without radicals. You may **not** assume that $b > 0$.

9(13 pts). Find all real solutions x to the given equation.

a. $\tan(3x) = \sqrt{3}$ b. $\cos^2 x + 3\cos x = 1 + \sin^2 x$

1.(Source: 4.2.15-32,4.2.more.1,4.4.3-18) Determine the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the x-axis and a line from the terminal point to the origin. Look for these two triangles. You were not required to rationalize denominators.

t	0	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$-\frac{17\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{17\pi}{2}$
quadrant	East Pole	I	II	III	II	South Pole
ref. number	0	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\sin t$	0	$\sqrt{3}/2$	$1/2$	$-1/2$	$1/\sqrt{2}$	-1
$\cos t$	1	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/2$	$-1/\sqrt{2}$	0
$\tan t = \frac{\sin t}{\cos t}$	0	$\sqrt{3}$	$-1/\sqrt{3}$	$1/\sqrt{3}$	-1	DNE
$\cot t = \frac{\cos t}{\sin t}$	DNE	$1/\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	-1	0
$\sec t = \frac{1}{\cos t}$	1	2	$-2/\sqrt{3}$	$-2/\sqrt{3}$	$-\sqrt{2}$	DNE
$\csc t = \frac{1}{\sin t}$	DNE	$2/\sqrt{3}$	2	-2	$\sqrt{2}$	-1

2.(Source: 4.1.28,31) $-64^\circ \frac{\pi}{180^\circ} = \frac{-16}{45}\pi$ radians.

3.(Source: 4.1.69,70) Arclength $= s = \theta r = 2 \cdot 3 = 6$ cm.

4a.(Source: 4.5.37) Use the half-angle identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ with $x = \frac{3\pi}{8}$ and get

$$\cos^2 \frac{3\pi}{8} = \frac{1}{2}\left(1 + \cos\left(\frac{3\pi}{4}\right)\right) = \frac{1}{2}\left(1 + \frac{-1}{\sqrt{2}}\right).$$

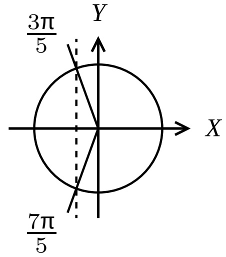
(We found $\cos\left(\frac{3\pi}{4}\right)$ in the table above.) Since $\frac{3\pi}{8}$ is in Quadrant I, we expect its cosine to be positive. Therefore take the positive root to find $\cos \frac{3\pi}{8} = \sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)}$.

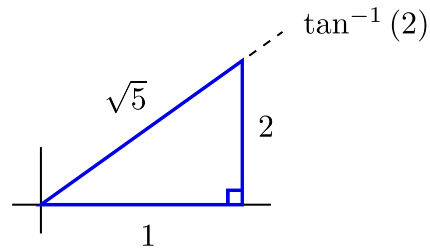
4b.(Source: 4.5.8) $\sin\left(-\frac{5\pi}{12}\right) = \sin(\frac{4\pi}{12} - \frac{9\pi}{12}) = \sin(\frac{\pi}{3} - \frac{3\pi}{4}) = \sin(\frac{\pi}{3})\cos(\frac{3\pi}{4}) - \cos(\frac{\pi}{3})\sin(\frac{3\pi}{4})$ (See table above for these sines and cosines.) $= \frac{\sqrt{3}}{2} \cdot \frac{-1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{-\sqrt{3}-1}{2\sqrt{2}}$.

4c.(Source: 4.7.2) $\tan^{-1}(\sqrt{3})$ is the unique angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent is $\sqrt{3}$. (We saw this angle in the table above.) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

4d.(Source: 4.7.more.2g) $\sin^{-1}\left(-\frac{1}{2}\right)$ is the unique angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ whose sine is $-\frac{1}{2}$, so $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

4e.(Source: 4.7.more.7m) $\theta = \tan^{-1}(2)$ is the angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent is 2. Since its tangent is positive, we know that θ is in Quadrant I. Use $\tan\theta = 2$ to label the horizontal and vertical legs of the triangle in Quadrant I. (See the figure on the right below.) Find the missing side ($= \sqrt{5}$) by the Pythagorean theorem. Then sine of θ is $y/r = 2/\sqrt{5}$.





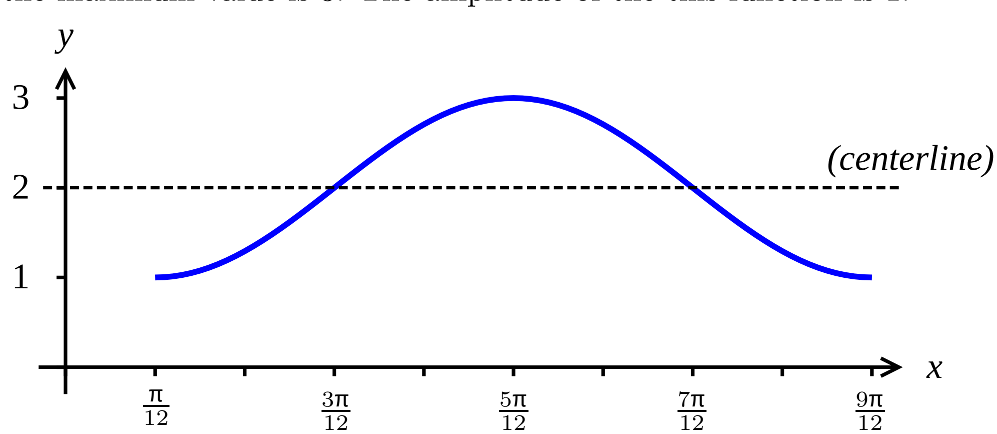
You can also solve this problem from the Pythagorean identity $\sec^2\theta = \tan^2\theta + 1 = 2^2 + 1 = 5$. Since θ is in Quadrant I, its secant is positive, and so $\sec\theta = \sqrt{5}$. Then $\sin\theta = \tan\theta\cos\theta = 2 \cdot \frac{1}{\sqrt{5}}$.

4f.(Source: 4.7.more.5q) $\frac{7\pi}{5}$ is in Quadrant III and its cosine is the x-coordinate of its terminal point. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$ is the angle θ in $[0, \pi]$ for which $\cos\theta = \cos\left(\frac{7\pi}{5}\right)$. Since the angle between the terminal side of $\frac{7\pi}{5}$ and the x-axis is $\frac{2\pi}{5}$, θ must be $\frac{3\pi}{5}$. (See the figure on the left above.)

5.(Source: 4.3.31-36,43) The function $y = 2 - \cos\left(3x - \frac{\pi}{4}\right)$ will go through one cycle when the angle inside the cosine goes from 0 to 2π.

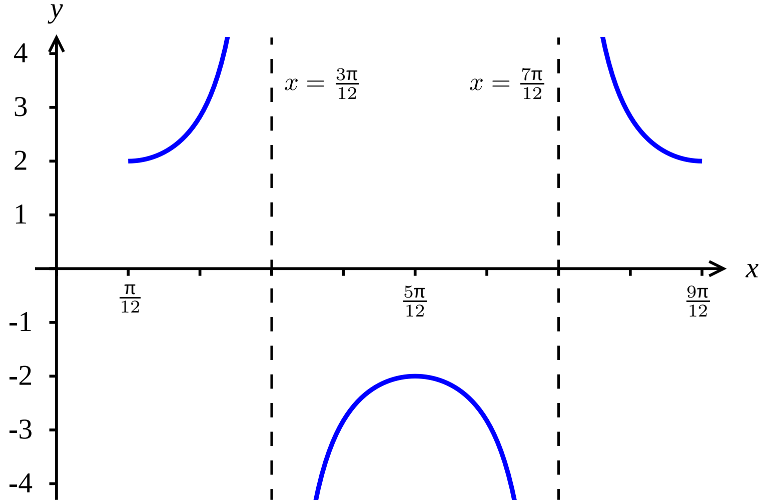
$$0 \le 3x - \frac{\pi}{4} \le 2\pi \qquad \frac{\pi}{12} \le x \le \frac{9\pi}{12}$$
$$\frac{\pi}{4} \le 3x \le 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$$

So, our cycle of the cosine will start at $x = \frac{\pi}{12}$ and end at $x = \frac{9\pi}{12}$.
The negative sign reflects the graph of cosine across the x-axis, and the 2 shifts the curve up two units, so that the new centerline of the curve is $y = 2$. The minimum value of y is 1, and the maximum value is 3. The amplitude of the this function is 1.

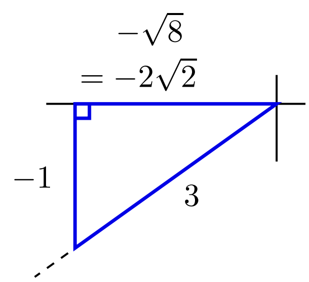


6.(Source: 4.4.39,41) $2\sec\left(3x - \frac{\pi}{4}\right)$ will go through one cycle when $\frac{\pi}{12} \le x \le \frac{9\pi}{12}$, as in Problem 5. The **period** is the length of this interval, or $\frac{8\pi}{12} = \frac{2\pi}{3}$.

$\sec\theta$ is the reciprocal of $\cos\theta$, so the secant is positive when the cosine is positive, negative when the cosine is positive, ± 1 when the cosine is ± 1, and has a vertical asymptote when the cosine has a zero. The 2 stretches the graph vertically.



7.(Source: 4.2.3, 4.4.23) Since $\frac{\pi}{2} < x < \frac{3\pi}{2}$ and $\sin x < 0$, we know that x is in Quadrant III. Use $\sin x = -1/3$ to label the vertical and hypotenuse of the triangle in Quadrant III. Find the missing side by the Pythagorean theorem.

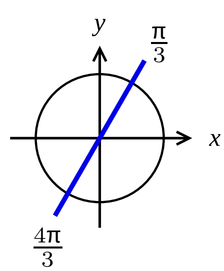


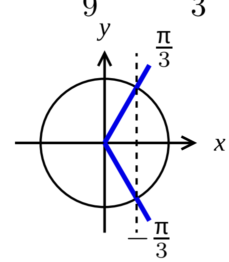
$$\sin x = -\frac{1}{3} \qquad \tan x = \frac{1}{2\sqrt{2}} \qquad \csc x = -3$$
$$\cos x = -\frac{2\sqrt{2}}{3} \qquad \cot x = 2\sqrt{2} \qquad \sec x = -\frac{3}{2\sqrt{2}}$$

8.(Source: 4.4.48) $\sqrt{b^2 + (b\tan\theta)^2} = \sqrt{b^2 + b^2\tan^2\theta} = \sqrt{b^2(1+\tan^2\theta)} = \sqrt{b^2\sec^2\theta} = |b\sec\theta| = |b|\sec\theta$. This last equality is due to the fact that $\sec\theta > 0$, because $-\pi/2 < \theta < \pi/2$. Note that we can't say that $\sqrt{b^2} = b$ unless we assume that $b \geq 0$.

9a.(Source: 4.8.30) $\tan(3x) = \sqrt{3}$ means that the terminal side of the angle $3x$ has slope $\sqrt{3}$. See figure on left below.

$$3x = \frac{\pi}{3} + 2\pi n \text{ or } \frac{4\pi}{3} + 2\pi n, \text{ and therefore } x = \frac{\pi}{9} + + \frac{2\pi n}{3} \text{ or } \frac{4\pi}{9} + \frac{2\pi n}{3}.$$





9b.(Source: 4.8.more.1p) Rewrite entirely in terms of $\cos x$, then get zero on one side and factor the other. $\cos^2 x + 3\cos x = 1 + (1 - \cos^2 x)$ implies $0 = 2\cos^2 x + 3\cos x - 2 = (2\cos x - 1)(\cos x + 2)$, which implies $\cos x = \frac{1}{2}$ or $\cos x = -2$. See figure on the right above. $\cos x = -2 < -1$ has no solutions, but $\cos x = \frac{1}{2}$ implies $x = \pm\frac{\pi}{3} + 2\pi n$.